Kinetics of Particle Growth in the Fluidized Bed Calcination Process

EARL S. GRIMMETT
Phillips Petroleum Company, Idaho Falls, Idaho

The fluidized bed calcination process involves spraying an aqueous solution of metal salts through pneumatic atomizing spray nozzles into a heated, gas-fluidized bed of granular solids where the salts are largely converted to the oxides. The free flowing granular solids produced continuously in the bed leave the reactor through an overflow pipe or a bottom draw-off line, and water vapor and volatile decomposition products leave with the fluidizing gas. Heat is furnished to the fluidized bed by heating the vessel walls (for small vessels) or by placing heat exchangers in the fluidized bed. A schematic diagram of the 2-ft.-sq. pilot fluidized bed calciner used to obtain some of the data in this paper is shown in Figure 1.

An important aspect in developing the fluidized bed calcination process for aluminum nitrate reactor fuel wastes (*1, 2, 3, 4, 5*) is control of the particle size in the bed. If particles in the bed become too large, fluidization becomes poor and heat transfer rates decrease to the point where bed caking will occur. On the other hand if the particles in the bed become too small, an excessive amount of the bed is entrained in and carried out of the calciner with the off-gas. A desirable mass median particle diameter range is from 0.3 to 0.8 mm., and process control requirements are such that neither can be passed without beginning to create difficult operational problems.

Previous studies of the mechanisms by which particles grow in a fluidized bed are not extensive because of the relatively recent use of the technique. Dunlop, Griffin, and Moser (*6*) studied particle size control and qualitative aspects of particle growth when coking heavy petroleum residua in a fluidized bed coker. They observed that normal particle growth can take place by virtue of laying down product coke on the individual particles or by agglomeration through the collision of particles containing tacky deposits of partially coked residuum, with the former being by far the most prevalent. They also concluded that particle growth occurs by deposition of a uniform thickness of coke on each particle regardless of size. No particle growth rates were reported however.

Lee et al. (*7*) studied the kinetics of particle growth in a fluidized bed where aluminum nitrate solutions were converted into granular particles of aluminum oxide. As did Dunlop et al., they concluded that the predominant particle growth mechanism is one of layer-by-layer deposition of newly calcined material on the particles in the bed and that the growth rate of a particle is proportional to its surface area.

Metheny and Vance (*8*) studied the control of particle size and particle growth in fluidized bed dryers, where solutions of ammonium sulfate, sodium hydroxide, calcium chloride, and sodium cyanide were converted into their respective solids. They concluded that the particle size distribution of the solid product is controlled by the size of the seed particle, by the adjustment of the ratio of the diameters of the dryer growth zone to its classifying zone, and by adjusting the liquid to solid (seed) feed ratio. No quantitative particle growth rates were reported by these investigators.

Philoon, Sanders, and Trask (*9*) studied the production of uranium trioxide by denitrating uranyl nitrate solutions introduced into a fluidized bed through a pneumatic atomizing nozzle. They concluded that the product particle size increased as the operating temperature of the calciner increased and decreased as the ratio volume of atomizing air to volume of liquid feed decreased. The concentration of the feed liquor and its rate of introduction and the fluidizing velocity all exhibited small effects on the product particle size. No particle growth rates were reported.

A study of the kinetics of particle growth and particle size distribution in the fluidized bed calcination process has been pursued at this laboratory since the initiation of the calcination program. Results of studies made during this program on particle growth kinetics are reported in this paper and are compared with those of Dunlop et al. (*6*) and Lee et al. (*7*). An equation for calculating the

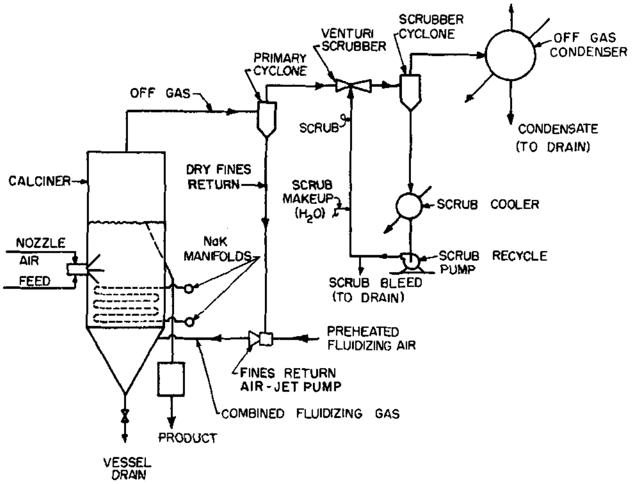

Fig.1. Schematic diagram of the 2-ft. sq. pilot plant fluidized bed calciner used to obtain some of the data in this report.

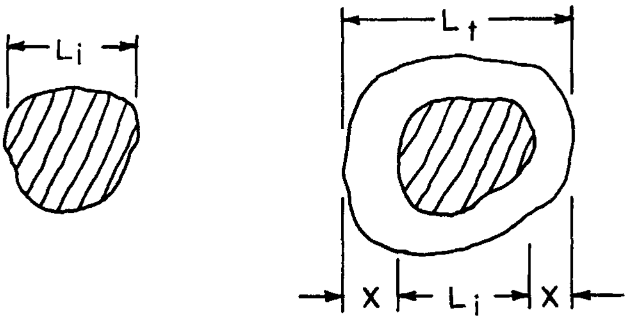

Fig. 2. How a particle increases in diameter when a layer of feed material of thickness x is deposited.

particle growth rate for the calcination process is presented.

THEORETICAL CONSIDERATIONS

Consider a fluidized bed consisting of particles of various sizes in which mixing is so efficient that a bed sample will have the same particle size distribution regardless of its location. If no other forces come into play, each particle in the bed over a significant period of time will spend the same amount of time in the feed spray zone. Furthermore if the rotation of each particle is completely random, then all of the surface area of each particle in the bed will be exposed to the feed spray for the same length of time. Since the total surface area in the bed is the sum of the surface area of each particle in the bed, all of the surface area in the bed will then be exposed to the feed spray for the same length of time.

Based on these assumptions the instantaneous rate at which feed material is deposited on the surface of each particle will be equal to the rate at which feed is deposited on the total surface in the bed, or

$$\frac{dx}{dt} = \frac{2(P/\rho)}{\sum_{i=1}^{i=\infty} A_i N_i}$$

where P is equal to the total mass rate of build up of calcined feed material on the bed (does not include that portion of the calciner feed which does not adhere to the bed particles), ρ is the density of the calcined feed material, A_i is the surface area of a particle in the i^{th} size interval, N_i is the number of particles in the i^{th} size interval, and the expression

$$\sum_{i=1}^{i=\infty} A_i N_i$$

is equal to the total surface area in the bed. The surface radial growth rate dx/dt can be constant only when the system is at steady state because all factors on the right-hand side of Equation (1) may change with time if operating conditions are changed.

The particle growth rate can be related to the radial growth rate as illustrated in Figure 2. Here L_i is the original effective particle diameter, x is the thickness of the material deposited over some period of time, and L_t is the effective diameter of the particle at the end of some growth period.

Since bed particles formed in this process are nearly spherical, an effective particle diameter is employed which is usually calculated as the arithmetic average of the measured major and minor axes (for single particles) or the arithmetic average of the adjacent screen openings (for groups of screened particles).

Let G be the particle diametral growth during time t. Then

$$G = L_t - L_i = (2x + L_i) - L_i = 2x$$

or the instantaneous average diametral particle growth rate g_a is

$$g_a = \frac{dG}{dt} = \frac{2dx}{dt} \tag{2}$$

When one combines Equations (1) and (2), the instantaneous average diametral particle growth rate can be expressed as

$$g_a = \frac{dG}{dt} = \frac{2dx}{dt} = \frac{2(P/\rho)}{\sum_{i=1}^{i=\infty} A_i N_i} \tag{3}$$

If the original assumptions prevail, Equation (3) will hold for a calcining or drying system of any size whose material density can be determined. On the other hand if some force acts on the particles so that the length of time a particle remains in the spray zone is dependent upon its size, then Equation (3) must be altered by including a function to take into account the preferential rate of growth of particles of one size over those of another size. As will be shown later data suggests that this function is a simple linear one, perhaps of the form $(A + BL)$ where A and B are arbitrary constants. If this function is the correct one and is combined with Equation (3), an expression for the instantaneous diametral growth rate of a particle of any size L is formed, or

$$g = \frac{2(P/\rho)}{\sum_{i=1}^{i=\infty} A_i N_i}(A + BL) \tag{4}$$

TECHNIQUES FOR DETERMINING PARTICLE GROWTH RATES

Two different methods of determining particle growth rates in fluidized bed coking or calcining systems have been reported in the literature. Dunlop, Griffin, and Moser (*6*) used as a starting bed coke having a known high ash content. As coking took place, particle growth occurred through the deposition of coke having a known low ash content. Near the end of a run a sample of the product coke was screened into different size fractions; the ash content of each size fraction was analytically determined. From this information the average particle growth rate over the time interval was determined.

Lee et al. (*7*) determined particle growth rates by using a radioactive tracer technique employing 65 to 100 mesh particles containing ruthenium-106 as traced seeds. Throughout a run many periodic samples of product were separated into different size fractions, and the activity of each size fraction was determined by its gamma activity. From this information the volume surface mean diameter of the particles in the bed could be determined as a function of time, and hence the mean diametral particle growth rate could be calculated.

In the present study three methods of determining particle growth rates were used. Data from two runs were obtained by measuring the growth between identifiable shells created in each particle by periodic introduction of a tracer material into the feed. Data from a third run were obtained by a chemical tracer adaptation of the method of Dunlop, Griffin, and Moser. Data from a fourth run were obtained by periodically measuring the growth of alumina around sand particles which made up the original bed.

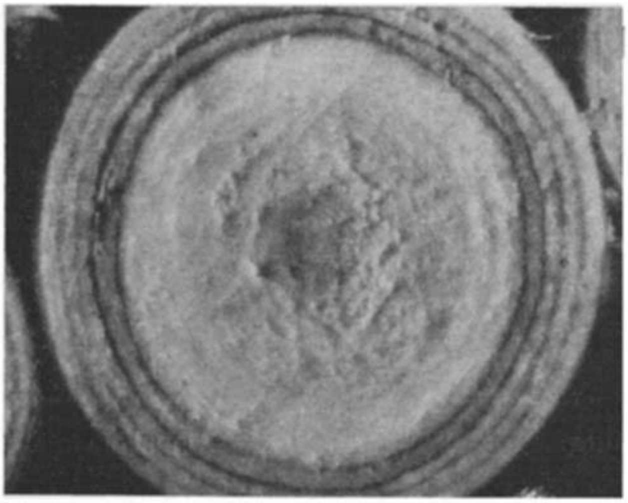

Fig. 3. Photomicrograph of sectioned calciner product particles showing growth rings.

Measuring Identifiable Growth Rings

One method of determining particle growth rate, and by far the most satisfactory one employed, is to measure directly the growth of large numbers of individual particles in the sample. This is made possible by periodically forming identifiable shells within the particles as the particles grow. When these particles are sectioned and examined under a microscope, the shells appear as rings which identify the time and shape (at least in cross section) of the particle when each shell was formed. Each ring in a particle must be identified with a particular time because, as will be shown later, not every particle examined contained all four rings. Figure 3 is a photomicrograph of a sectioned particle showing the identifiable rings formed within a particle by this technique.

Identifiable shells, or rings, can be formed in the alumina particles by adding a soluble metal nitrate salt to the calciner feed for a short period of time. When this salt is calcined with the feed at the calciner bed temperature, a colored oxide is formed which is visible in the white product aluminum oxide. Nitrate salts of iron, nickel, copper, and chromium form identifiable rings of red, black, blue-green, and yellow, respectively, in the product particles. Each spiked feed is introduced into the calciner for a period of 15 min.; the concentration of each spike salt is that which is soluble in the regular feed solution at room temperature. At least 8 hr. is allowed to elapse between spike additions.

At the end of a run a sample of the product is screened into size fractions, and a sample of each size fraction is mounted in clear plastic. The mounted sample is ground and polished until the particles are exposed in cross section at or near their diameters. The various colored rings are exposed, and the effective diameter of the particle at the time each ring was formed is determined by microscopically measuring its major and minor axes.

From these measurements the effective particle diameter can be plotted as a function of time. The slope of this curve at any time is the instantaneous growth rate of the particle at that time. If these curves are plotted for different sized particles, then the growth rates of different sized particles measured at identical times will reflect the effect of particle size *per se* but not the effect of other factors which may be changing with time. Herein lies the outstanding advantage of this technique.

This method has one disadvantage. Because the growth process which occurs on the particles in the fluidized bed is a random one, large numbers of particles must be examined if meaningful data are to be obtained, and the data must be statistically treated. Although it is technically superior, more time and money are required to obtain data by this technique than by either of the other two techniques.

The Chemical Tracer Technique

At an initial time t_0 a chemical compound which can be quantitatively analyzed at low concentrations is introduced into the calciner feed system. Calcined feed containing the chemical tracer at a known concentration forms into layers around the particles in the bed. Samples of the product are periodically separated into size fractions, and each size fraction is analyzed for the chemical tracer.

A particle with an effective diameter L_f when removed from the calciner at time t_f is composed of an inner core of effective diameter L_i which contains no chemical tracer and an outer layer of thickness $(L_f - L_i)$ which contains the chemical tracer at a concentration of x_f. The total concentration of the chemical tracer in the particle is x_m. A material balance of the chemical tracer can be made around the particle, or

$$\underset{\text{(core)}}{\frac{0 \cdot \pi L_i^3 \rho}{6}} + x_f \cdot \underset{\text{(outer layer)}}{\frac{\pi (L_f^3 - L_i^3) \rho}{6}} = x_m \cdot \underset{\text{(whole particle)}}{\frac{\pi L_f^3 \rho}{6}} \qquad (5)$$

If the particle density is assumed uniform throughout the particle, Equation (5) can be rearranged to give

$$L_i = L_f \sqrt[3]{1 - \frac{x_m}{x_f}} \qquad (6)$$

Equation (6) can be used to calculate the increase in the effective diameter of the original particle over the period of time chosen, and from this value the average diametral particle growth rate can be calculated for this time.

This method has disadvantages which are not at first evident. Errors introduced while screening the sample, and sampling errors, seriously decrease the precision of the data. Analytical errors in determining the concentration of the tracer material in the samples can introduce particularly large errors in the final data. The most striking disadvantage of this method however is that only an average growth rate over a given time interval can be determined. Even with these disadvantages this method has contributed important information concerning the growth of particles in the fluidized bed calcination process.

Measuring the Feed Buildup on Sand

If a calciner run is started with an initial bed of sand, the progress of particle growth can be followed by observing the buildup of calcined feed material around the sand particles. Figure 4 shows a photomicrograph of cal-

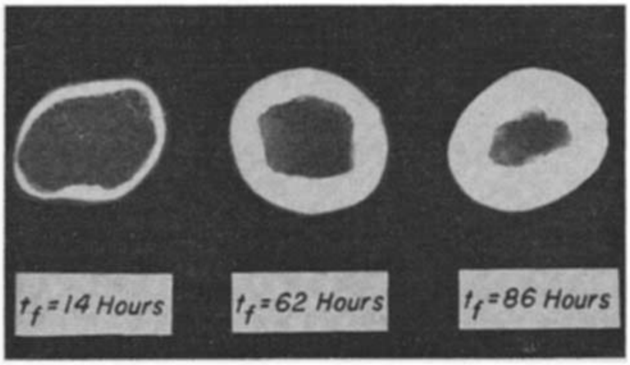

Fig. 4. Photomicrograph of sectioned calciner product particles showing the build up of calcined alumina around sand grains.

TABLE 1. CALCINER RUN CONDITIONS USED FOR PARTICLE GROWTH RATE EXPERIMENTS

	Runs			
	A	B	C	D
Calciner*	24 in.	24 in.	24 in.	12 in.
Bed temperature, °C.	500	425	400	400
Feed rate, liter/hr.	40	90	80	10
Bed weight, kg.	200	154	230	79
Average product overflow rate, kg./hr	3.83	7.05	5.50	0.51
Theoretical product rate, kg./hr.	4.30	7.25	6.05	0.73
Product particle density, g./cc.	1.14	1.41	1.48	1.79
Nozzle air-to-liquid volume ratio†	306	400	450	250
Superficial fluidizing velocity, ft./sec.**	1.0	1.0	1.0	1.0
Type of feed nozzles	‡	***	***	****
Length of test period on run, hr.	20	86	32	133
Feed composition:				
Aluminum nitrate, molar	1.95	1.50	1.29	1.29
Nitric acid, molar	1.25	1.26	2.34	2.34
Sodium nitrate, molar	0.089	0.088	0.078	0.078
Mercuric nitrate, molar	0.006	0.006	0.015	—
Boric acid, molar	—	—	0.050	0.010

* 24 in. refers to the 2-ft.-sq. calciner, and 12 in. refers to the 12-in.-diameter calciner.

† The air volume is determined at the metered temperature and vessel pressure.

** The superficial fluidizing velocity is determined on the basis of the empty cross section of the calciner at the calciner bed temperature and vessel pressure and includes only those gasses entering through the bed support plate.

‡ Four Spraying Systems Company type ¼ J (set up No. 4) air atomizing nozzles.

*** One Spraying Systems Company type ½ J (set up No. 70) air atomizing nozzle.

**** One Spraying Systems Company type ¼ J (set up No. 48) air atomizing nozzle.

ciner product where alumina has been deposited around the sand particles which made up the original calciner bed.

Just as in the growth ring technique the growth of a particle during an interval of time determines for that interval the average particle growth rate. Its growth can be determined by microscopic measurements made directly from sectioned product particles or from measurements taken from photomicrographs of sectioned product particles. In contrast to the growth ring technique only two circumferential places in a particle can be related to a definite time. The interface betwen the sand particle and the deposited feed material is identified with the time of initial feed introduction, and the outside of the particle is identified with the time the particle was removed from the calciner.

A disadvantage of this technique is similar to that of the chemical tracer technique; only the average growth rate of a particle between two intervals of time can be determined. Even when a large numbr of samples are analyzed throughout a run, any changes in particle growth rate during the run cannot be determined.

EXPERIMENTAL

Experimental runs were made in a 2-ft.-sq. calciner (*4, 5*) and a 12-in. diameter calciner. Feed for both calciners was introduced into the beds through similar but different sized pneumatic atomizing nozzles. The 2-ft.-sq. calciner was heated by circulating sodium potassium eutectic at temperatures as high as 1,400°F. through a heat exchanger tube bundle placed directly in the fluidized bed of particles; the 12-in.-diameter calciner was heated by tubular electrical heaters placed in the fluidized bed of particles. Feed capacities of the two units were markedly different; the larger unit had a maximum feed capacity of 150 liters/hr., while the smaller unit had a maximum feed capacity of 12 liters/hr. Product from each unit continuously overflowed from the top of the fluidized bed into a product catch pot which was periodically emptied.

Data from four runs are included in the correlations presented herein. Three of these runs were made in the 2-ft.-sq. calciner (Runs A, B, and C) in conjunction with other experiments of higher priority. Another run (Run D) was made in the 12-in.-diameter calciner for the express purpose of obtaining particle growth rate data.

Data from Run A were obtained by using the chemical tracer technique, employing iron nitrate tracer dissolved in the regular feed solution at an iron concentration of 0.288 g./cc. Run B data were obtained by measuring the build up of calcined feed on sand; data from the other two runs were obtained by using the growth ring technique. Operating conditions for each run are summarized in Table 1. Boric acid was used during Runs D and C to inhibit the formation of alpha alumina in the product; mercuric nitrate was not used in Run D because it was found by that time to have little effect on the product properties in the concentrations found in the actual radioactive waste solutions.

Table 1 indicates the length of time over which the four experimental runs were made. With the exception of Run D the time of the experiments varied because the particle growth rate experiments were conducted during available time which was not always optimum for growth rate studies.

Table 1 also lists the bed weights, product rates, and product particle densities actually obtained during each of the four runs. The differences between the actual overflow product rates and the rates calculated from the feed rates and feed concentrations are caused by elutriation of fines from the calciner. It was not possible to distinguish on these runs between those elutriated fines which never participate in the particle growth phenomenon, that is those feed spray droplets which never adhere to a particle and become spray dried, and those fines which are produced as a result of attrition of the bed particles. Arbitrarily the quantity *P* used in calculations involving particle growth rate has been assigned the value of the average overflow product rate throughout a given run, which is strictly correct only if attrition of bed particles is negligible.

EXPERIMENTAL RESULTS

The method of analyzing the data obtained by the tracer ring method is illustrated in Figure 5, where the mean effective particle diameter for each screen cut is plotted against time. Zero time is identified in the sectioned particles by the innermost red ring. Each succeeding ring identifies the particle diameter at the time the ring was formed. At least twenty particles were measured in each sample (a confidence limit for particle growth rates

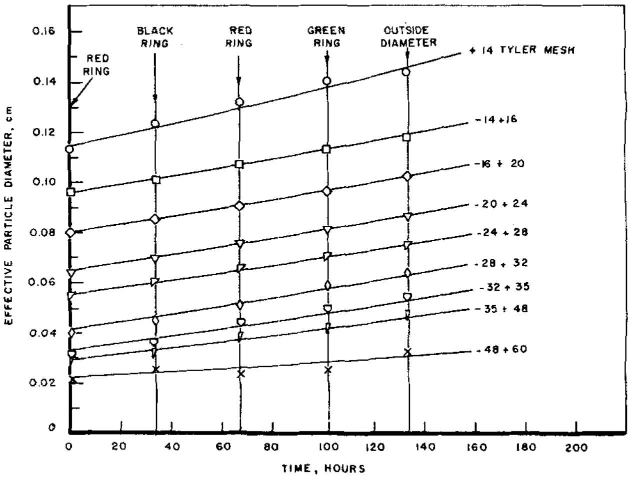

Fig. 5. Summarized particle growth rate data from Run D.

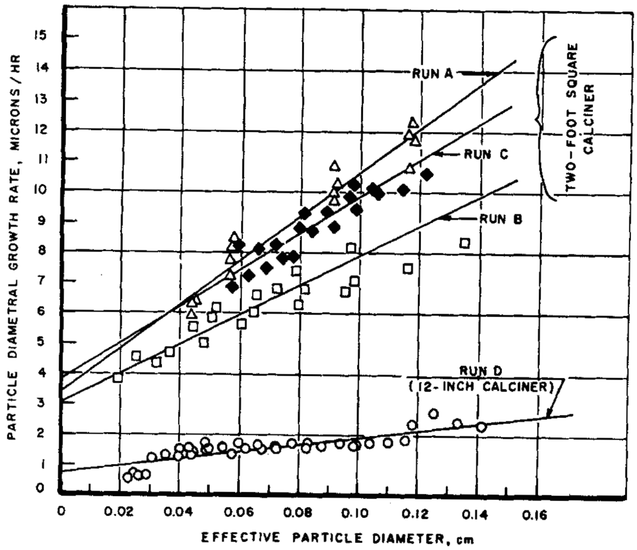

Fig. 6. Plots of Equation (4) with least-square values for A and B for each run.

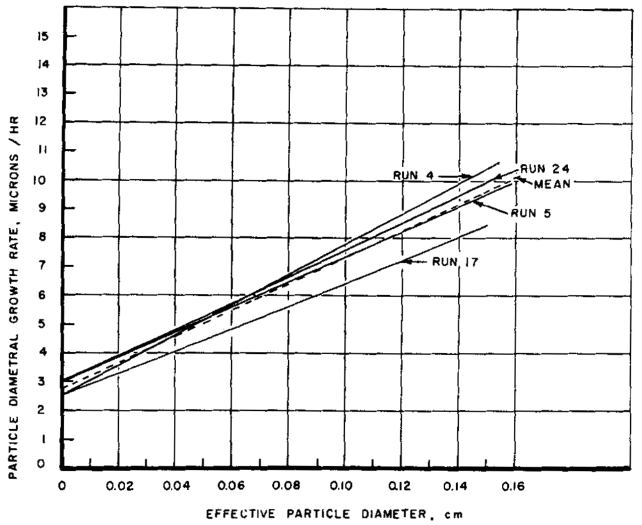

Fig. 7. A comparison of particle growth rate curves for each run with the mean curve for all runs. All curves are normalized to $g_a = 5\mu$/hr.

of 95% requires that twenty particles be examined), although more of the larger particles contained all four rings than did the smaller particles. Thus when the effective particle diameters of the rings in the $-48 + 60$ mesh sample for example were averaged to obtain the data points on Figure 5, the smaller number of data samples making up the average ring diameters introduced considerable error into the plotted data. It is evident then, that the data obtained from the larger sized particles should be better than that obtained from the smaller sized particles. The particle diametral growth rate is equal to the slope of the curve at any point on the curve. Growth rates using the other measuring techniques were determined in similar but not indentical ways.

After effective particle diameters and particle growth rates at arbitrarily chosen times during each run were obtained, the data were plotted as shown on Figure 6. Constants A and B from Equation (4) were determined for each run by treating the data by the method of least squares.

The validity of the form of Equation (4) can now be evaluated. If the particle growth rate in this process is not a function of the particle diameter, then the value for B would have to be zero and the value for A would have to be unity. Obviously this is not the case, and therefore one must conclude that the particle growth rate is a function of the particle diameter.

Next the generality of Equation (4) for the conditions studied must be determined. To be general for this process the constants A and B, respectively, must be equal for all of the runs. The experimental data show that with a mean value of 0.542 and a standard deviation of 0.046, 95% of the A values fall between 0.452 and 0.632. Likewise with a mean of 9.12 and a standard deviation of 1.14, 95% of the B values fall between 6.88 and 11.36. Although the A and B values are not absolutely constant, their statistical approach to constancy is significant. Within these limits Equation (4) can be considered general for the process.

It is not presumed that the function $(A + BL)$ is the only one which will correlate the data. Undoubtedly such a function can and should be developed from theoretical considerations. Before such a task is undertaken however a better understanding of the nature of the interactions between the fluidized bed and the pneumatic atomizing feed nozzle should be obtained as well as additional growth rate data from experimental runs designed for this purpose only.

It is interesting to compare curves of effective particle diameter vs. growth rate calculated from the A and B values for each of the four runs and for the mean A and B values of the four runs. This comparison is shown in Figure 7. For this illustration an arbitrary value of 5 μ/hr. was assigned to g_a. The curves for each of the four runs deviate from the mean curve no more than ±12%, even though the operating conditions, the calciner sizes, and the number and sizes of the feed nozzles varied widely from run to run.

Dunlop et al. (*6*) did not relate clearly the actual instantaneous particle growth rate taking place in the fluidized bed coker with the properties of the product coke, with the feed rate of the residuum to the coker, or with the weight and particle size distribution of the bed in the coker. Lee et al. (*7*) empirically related these factors to the particle growth rate taking place in their calciner but did not show, as does Equation (3), the simple fundamental relationship which exists between the particle growth rate and these factors for fluidized bed calcining systems. No previous investigator allowed for the possibility that nonrandom behavior of particles might cause particles of one size to grow faster than particles of another size. These two points are discussed further in the following paragraphs.

Equation (3) allows calculation from fundamental considerations of an average particle growth rate for a particular fluidized bed calciner operating at some specified conditions even if some of the particles in the bed grow faster than do other particles in the bed. Since the particle growth rates obtained experimentally by Lee et al. (*7, 10*) are average rates for all sizes of particles on each run, a comparison of their data and Equation (3) is of interest. Enough data from their Run PS-19 are available to make such a comparison. With a bed weight of 900 g., a particle density of 1.77 g./cc., a product rate of 81.6 g./hr., and a bed surface area of 78,300 sq. cm. at the start of the run and 48,500 sq. cm. at the end of the run the average particle growth rate as calculated by Equation (3) was 11.7 μ/hr. at the beginning of the run and 18.7 μ/hr. at the end of the run, averaging 15.2 μ/hr. for the entire run. The average particle growth rate for the run as determined by their radioactive tracer technique was 17.5 μ/hr., a reasonable agreement.

Equation (4) is based on the same simple relationships which exist between the calciner operating variables and the average particle growth rate in the bed, but in addition

TABLE 2. PER CENT OF PARTICLES IN THE SCREEN SIZED SAMPLES INDICATED WHICH SHOW EVIDENCE OF HAVING PASSED THROUGH THE FEED SPRAY ZONE DURING A 15-MIN. FEED SPIKE PERIOD

Tyler screen size	Red ring	Black ring	Red ring	Green ring
−12 +14	70	65	74	99
−14 +16	50	69	75	94
−16 +20	42	63	65	84
−20 +24	58	50	38	80
−24 +28	42	58	58	62
−28 +32	30	58	34	30

it incorporates the effect of particle size on the growth rate of individual particles in the bed. It is entirely possible that in some systems no such effect of particle size exists, and in those cases Equation (4) simplifies to Equation (3). However the experimental results from this study show convincingly that for the systems studied particle growth rate is a function of particle size at any given time.

In all probability the factor which determines whether the calcining system is represented by Equation (3) or Equation (4) is whether the motion of all particles in the fluidized bed is completely random. In considering particle growth in a well-fluidized bed it is usual to assume that events occur in a random way. However the data obtained in this study show that bed particles do not behave in a totally random fashion as they pass through the feed spray zone. For some reason the larger bed particles remain in this zone for a longer period of time over any significant period of time than do the smaller bed particles.

This departure from random behavior might be due to the way in which feed is introduced into the fluidized bed of particles. Liquid feed is atomized externally to the nozzle, but in the bed, by high velocity air which issues from the nozzle through an annular orifice. This orifice is concentric to the liquid orifice through which the feed leaves the nozzle. It is possible that this high velocity air forms a protective envelope or barrier around the spray zone which gives a selectivity to the penetration of particles into it; the larger particles seem to penetrate easier and further into the spray zone than do the smaller ones.

Further proof of this selectivity is shown in Table 2, which lists the percentage of the particles in various size ranges exhibiting evidence of having passed through the spray zone during each of the 15-min. intervals when the tracer materials were being introduced into the calciner with the feed. These data were obtained from Run C and again show strikingly that larger sized particles in the calciner bed penetrated into the spray zone more easily than the smaller ones.

CONCLUSIONS

1. Three methods for determining the particle growth rate in a fluidized bed calcination process have been investigated. A tracer growth ring technique is the technically superior method but has the disadvantage of being more costly and time consuming.

2. Runs were made in two calciners the size and capacity of which differed markedly. Three different sized feed nozzles were used during these runs. Even for this wide range of calciner operating conditions the particle growth rate was found to be a function of particle diameter as well as of the feed rate, of the surface area in the bed, and of the solid product density. For any given set of operating conditions the larger bed particles exhibit a higher growth rate than do the smaller bed particles.

3. The growth rate of any particle in the fluidized bed calcination process can be calculated by using Equation (4), where the best values for the arbitrary constants A and B are 0.542 and 9.12 cm.$^{-1}$, respectively, over the wide range of conditions studied.

ACKNOWLEDGMENT

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NOTATION

A_i = surface area of a particle in the i^{th} size interval, sq.cm.
A, B = constants appearing in general growth rate equation, A dimensionless, and B equal cm.$^{-1}$
G = particle diametral growth during any interval of time, cm.
g = particle growth rate, specific for a particle of any size, cm./hr.
g_a = instantaneous average diametral particle growth rate in bed, equal to

$$\frac{2(P/\rho)}{\sum_{i=1}^{i=\infty} A_i N_i}$$

L = effective particle diameter, cm.
L_o = effective particle diameter at zero or initial time, cm.
L_f = final effective particle diameter, cm.
L_t = effective particle diameter at time t
L_i = initial effective particle diameter, cm.
N_i = number of particles in the i^{th} size interval
P = equivalent solid feed rate = only that portion of the calciner feed which is deposited on the surface of the particles in the bed, g./hr.
t = time
t_f = time at some point during a run
t_o = starting time of a run
x = radial growth on particle surface area, cm.
x_f = concentration of chemical tracer in solid feed, g./cc.
x_m = concentration of chemical tracer in a particle at time t_f, g./cc.
ρ = density of the solid feed material being deposited around the particles in the bed, g./cc.
σ = standard deviation

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